Modeling the Steering Time Difference between Narrowing and Widening Tunnels

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ABSTRACT

The performance of trajectory-based tasks is modeled by the steering law, which predicts the required time from the index of difficulty (*ID*). This paper focuses on the fact that the time required to pass through a straight path with linearly-varying width alters depending on the direction of the movement. In this study, an expression for the relationship between the *ID* of narrowing and widening paths has been developed. This expression can be used to predict the movement time needed to pass through in the opposite direction from only a few data points, after measuring the time needed in the other direction. In the experiment, the times for five *ID*s were predicted with high precision from the measured time for one *ID*, thereby illustrating the effectiveness of the proposed method.

Author Keywords

Steering law; graphical user interfaces; pointing; human performance; modeling; motor control.

ACM Classification Keywords

H.5.2. [Information interfaces and presentation]: User Interfaces - Input devices and strategies.

INTRODUCTION

Modeling human movement is a core theme in the field of human–computer interaction (HCI). Well-known models include Fitts' Law [9], which predicts the time required for a one-dimensional pointing task, and a refined model for two-dimensional tasks by Accot and Zhai [5]. Others include modeling of the time taken to pass through start and end lines [4], or the time to navigate a long, narrow path, known as the steering law [1]. All of these depict a linear relationship between the index of difficulty (*ID*) and the movement time (*MT*) of a task.

A paper by Accot and Zhai [1], who derived the steering law, proposes a model which predicts the navigation time for a narrowing tunnel, such as that shown in Figure 1a, and shows the high degree of accuracy of that model through an experiment. In a revalidation experiment, the authors discovered that the navigation time decreased when moving in the opposite direction through a narrowing tunnel such as that shown in Figure 1b.

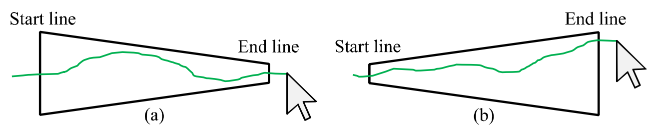

Figure 1. Steering through (a) a narrowing tunnel and (b) a widening tunnel.

To our knowledge, there is no research that investigates whether or not there is any kind of relationship between the navigation direction (narrowing or widening) and the *MT*. For example, if the *ID* were larger for the narrowing direction when compared with the widening direction by a fixed value, or a fixed ratio, then predicting the navigation time for the widening direction would be a simple task.

In this context, this study derives the theoretical relationship between the navigation time for narrowing and widening directions and then experimentally tests it. The contribution of this study can be summarized by the following two points:

- Derives the theoretical relationship between navigation direction and the *ID*.
- Verifies the validity of the relationship through an experiment, and shows that it is possible to predict–with a high degree of precision–the navigation time in one direction using the navigation time in the other direction.

RELATED WORK

In graphical user interface (GUI) environments, movement that is aligned with a specified trajectory is often desired. Selecting a particular item from a hierarchical menu is one example of this. In order to model this kind of two-dimensionally constrained action, Accot and Zhai proposed the steering law [1]. Their experiment was carried out using an indirect control method, in which the cursor on the display was controlled by a desktop stylus through three types of paths: a constant-width linear path, a narrowing linear path, and a widening helix. For each of the paths, their model was found to have a high degree of consistency. Its applicability has also been shown for constant-width

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loop paths [2, 3]. Furthermore, this law has been shown to hold in various environments, with experiments having been carried out under conditions such as changes in device [2], changes in scale (CD gain) [3], addition of a fisheye lens effect [10], hover operations, in which the stylus was held above the input pad [11], changes in cursor size [16], and changes in the friction of the stylus input pad [18]. The applicability of the model has also been shown in GUI environments with more than two dimensions, such as three-dimensional input [7, 14], and a driving simulator in which the cursor was replaced by a car [20].

Revision of the model is also being discussed. When the width of the path W becomes too wide, the ratio of the width used in practice (i.e., the effective width) decreases. Taking this into account, a model that calculates the *ID* has been proposed [12]. In this revised model, it has been shown that the model can be applicable if the task is continued and the effective width calculated even when the cursor leaves the path. Revisions which enable the prediction of navigation time through a path which includes corners has also been investigated [17], and models which hypothesize even more complex tasks are being constructed.

Methods for predicting the operation time of tasks which combine pointing and steering have also been proposed [8, 13]. For example, in order to include an operation for using a hierarchical menu that involves pointing at an item at the beginning and one at the end, it is important to investigate whether it is possible to predict the operation time by combining existing models. Further, experiments to verify the error rate when constraints are placed on operation time have been undertaken [19], and a trade-off between time and accuracy has been shown for steering tasks.

DERIVATION OF THE STEERING LAW MODEL AND *ID* DIFFERENCES DUE TO NAVIGATION DIRECTION

This section provides an overview of the method for deriving models for a constant-width linear path and for a narrowing linear path from Accot and Zhai's model [1], and considers reasons why navigation direction may cause differences in operation time.

Method for Deriving the Steering Law Model

Constant Width Linear Tunnel

When, as illustrated in Figure 2a, two line segments of width W are established at a distance A from each other, the time (MT_1) to cross these segments has been shown in experiments to be:

$$MT_1 = a + b \times ID_1,\ ID_1 = \log_2\left(\frac{A}{W} + 1\right) \tag{1}$$

where a and b are constants determined by the experiment. Based on this, placing line segments of width W between the original line segments, to divide the distance A into N segments, as shown in Figure 2b, will result in the ID_N for navigating the entire distance A being:

$$ID_N = N\log_2\left(\frac{A}{NW} + 1\right) \tag{2}$$

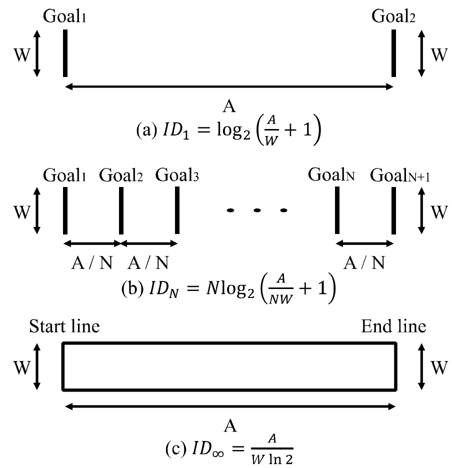


Figure 2. *ID* of a linear tunnel with constant width.

When $N \to \infty$, as in Figure 2c, $ID_\infty = A/(W \ln 2)$. Merging ln 2 into the fixed number b yields:

$$MT_\infty = a + b \times ID_\infty,\ ID_\infty = \frac{A}{W} \tag{3}$$

Accordingly, the time MT for navigating through an infinite number of line segments between the beginning and end segments is:

$$MT = a + b \times \frac{A}{W} \tag{4}$$

As this is the same as navigating through a tunnel of constant width, the time required for the steering task of navigating through a tunnel of width W and length A can be expressed as Equation 4.

Narrowing Tunnel

As shown in Figure 3a, the navigation time MT_{NT} for a tunnel of length A that narrows from width W_L to W_R can be calculated based on Equation 4. As shown in Figure 3b, dividing the tunnel into infinitesimal sections dx, and viewing the task as one of steering continuously through those short distances, allows MT_{NT} to be expressed as:

$$MT_{\mathrm{NT}} = a + b \times ID_{\mathrm{NT}} \tag{5}$$

$$ID_{\mathrm{NT}} = \frac{A}{W_R - W_L}\ln\frac{W_R}{W_L} \tag{6}$$

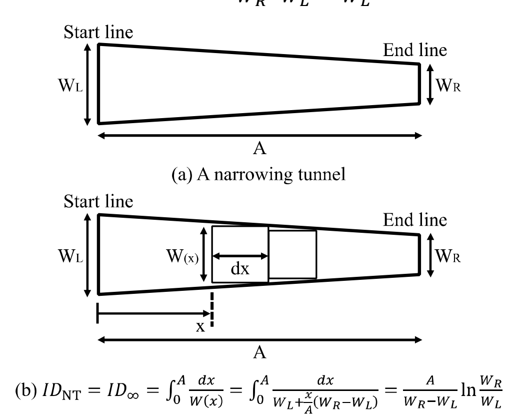


Figure 3. *ID* of a linear narrowing tunnel.

Accot and Zhai [1] define MT_{NT} as the time taken to pass through a narrowing tunnel, and investigate the accuracy of the model by experiment only for the case of moving in the narrowing direction.

Moving in the Widening Direction

We focus on the fact that the movement time MT_{WT} for moving in the widening direction is the same as Equation 5 and 6, for moving through a narrowing tunnel. Defining ID_{WT} as the difficulty of navigating Figure 3a from right to left yields:

$$ID_{WT} = \frac{A}{W_L - W_R} \ln \frac{W_L}{W_R} = \frac{A}{W_R - W_L} \ln \frac{W_R}{W_L} \quad (7)$$

and thus we obtain $ID_{WT} = ID_{NT}$. However, as it does, in actuality, take less time to navigate in the widening direction, it may be that some aspect has been overlooked in this method of derivation.

In our review of the literature, we were unable to find an experiment in which the direction of navigating a tunnel with changing width is changed. We inferred from this that the fact that a difference in operation time arises has not previously been discovered.

While Accot and Zhai's original experiment [1] focuses on a widening helix, it does not investigate the case of the narrowing direction, and therefore does not constitute a comparison of different directions of movement within the same shape.

ID FOR PREDICTING THE *MT* FOR THE OPPOSITE DIRECTION

With regard to the pointing task, it is known that the target location is reached by cycling through multiple accelerations and decelerations [6, 15]. It was our supposition that the steering task for a narrowing tunnel does, in a similar fashion, involve cycling through corrections of direction and speed, as illustrated in Figure 4. It is not certain whether movement corrections, such as those in Figure 4, arise when navigating a specific distance or on a time basis. However, in the Accot and Zhai's proposed model [1] where $N \to \infty$, this means that the number of corrections is infinite, and the difference in difficulty arising according to the navigation direction is not considered.

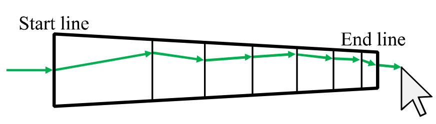


Figure 4. Steering through a narrowing tunnel with small movement corrections.

The degree of acceptable cursor slippage in a given infinitesimal section (for example, the vertical slippage in Figure 5) is not, in fact, fixed as Accot and Zhai propose, but depends on the width of the end line of that infinitesimal section. The reason for this is that since the cursor possesses velocity in the direction of the end line as soon as it enters the infinitesimal section, it is difficult to suppose that it will slip as far as the maximum width of the start line at the moment it crosses the start line.

Accordingly, as opposed to the case shown in Figure 5a, where for movement in the widening direction it is acceptable for the cursor to slip to the full width of the wide end of the infinitesimal section (i.e., toward the end line), for the narrowing movement shown in Figure 5b, it is not possible to use the entire wide end (i.e., the start line) owing to the momentum when crossing it, and thus the navigation difficulty increases. Of course, as the width of the start line for the infinitesimal section in Figure 5a is narrower than that in Figure 5b, it can be assumed that the time taken to enter the widening infinitesimal section would be longer. However, when taking the entire tunnel into account, the most difficult point of navigation in a widening tunnel is the start line. Therefore, the time taken for the start line, where the finest control is required, is not included in the recorded time needed for the steering task. Accordingly, the hypothesis of this study is that the overall navigation time is shortest for the widening tunnel ($MT_{WT} < MT_{NT}$). Of course, if the infinitesimal section distance dx is decreased infinitely, then Accot and Zhai's model will be consistent, but assuming that the number of corrections made during movement is limited, a difference in difficulty will arise according to the direction of navigation.

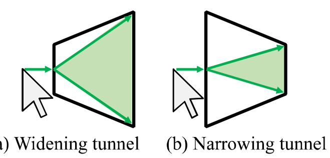
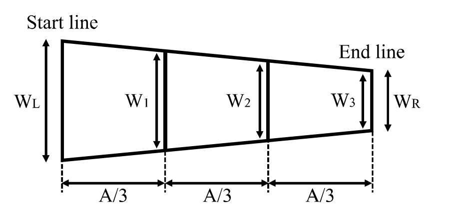


Figure 5. Acceptable error movement in an infinitesimal path.



(a) $ID_{NT(3)} = \frac{A/3}{W_1} + \frac{A/3}{W_2} + \frac{A/3}{W_3} = \frac{A}{2W_L + W_R} + \frac{A}{W_L + 2W_R} + \frac{A}{3W_R}$

(b) $ID_{WT(3)} = \frac{A/3}{W_1} + \frac{A/3}{W_2} + \frac{A/3}{W_3} = \frac{A}{3W_L} + \frac{A}{2W_L + W_R} + \frac{A}{W_L + 2W_R}$

Figure 6. Steering through a narrowing tunnel with three movement corrections.

More specifically, assuming, for example, that three corrections occur within the path length (i.e., N = 3) at regular distance intervals, the degree of difficulty $ID_{NT(3)}$ for navigating through a narrowing tunnel as illustrated in Figure 6a would be:

$$ID_{\mathrm{NT}(3)} = \frac{A/3}{(2W_L+W_R)/3} + \frac{A/3}{(W_L+2W_R)/3} + \frac{A/3}{W_R}$$
$$= \frac{A}{2W_L+W_R} + \frac{A}{W_L+2W_R} + \frac{A}{3W_R} \quad (8)$$

where the time taken for the start line is not included. In contrast, the degree of difficulty $ID_{\mathrm{WT}(3)}$ for navigating through a widening tunnel as in Figure 6b would be:

$$ID_{\mathrm{WT}(3)} = \frac{A}{3W_L} + \frac{A}{2W_L+W_R} + \frac{A}{W_L+2W_R} \quad (9)$$

Therefore, the difference in difficulty $ID_{\mathrm{Gap}(3)}$ yields:

$$ID_{\mathrm{Gap}(3)} = ID_{\mathrm{NT}(3)} - ID_{\mathrm{WT}(3)}$$
$$= \frac{A}{3W_R} - \frac{A}{3W_L} = \frac{A(W_L-W_R)}{3W_LW_R} \quad (10)$$

The difference in difficulty with the number of segments generalized as N–$ID_{\mathrm{Gap}(N)}$–is thus:

$$ID_{\mathrm{Gap}(N)} = \frac{A(W_L-W_R)}{NW_LW_R} \quad (11)$$

That is, N plays the role of the coefficient that determines the degree to which the path parameters are reflected in the difference in difficulty.

Up to this point, the degree of difficulty has been calculated by dividing distance A into N equal segments. In reality, however, corrections do not occur at regular distance or time intervals. Rather, they should occur according to the degree of change of width, of distance A, or of the operation device. Accordingly, replacing the number of equal partitions N with a free weight [5] k (> 0), determined by the experimental conditions, to Equation 11 yields:

$$ID_{\mathrm{Gap}(k)} = \frac{A(W_L-W_R)}{kW_LW_R} \quad (12)$$

which is the difference in difficulty of the steering task attributable to the navigation direction. As this equation has a different structure than the *ID* of the model proposed by Accot and Zhai (Equation 6), this implies a different relationship to the fixed value or fixed ratio.

As $k \to \infty$ in Equation 12, $ID_{\mathrm{Gap}(\infty)} \to 0$, which conforms with Accot and Zhai's derivation method which assumes that the navigation direction does not have any relationship to the degree of difficulty. This implies infinite corrections and means that the difference in difficulty due to the navigation direction is not adequately expressed. Conversely, as $k \to 0$, $ID_{\mathrm{Gap}(0)} \to \infty$, which is also inappropriate for the difference in difficulty. Accordingly, the most appropriate value for k (i.e., $0 < k < \infty$) should be used to express the difference in difficulty.

Two ways to use the difference in difficulty ID_{Gap} may be:

- Correct the navigation difficulty for the narrowing tunnel ID_{NT} to $ID_{\mathrm{NT}} + ID_{\mathrm{Gap}}$ and predict the *MT*.
- Correct the navigation difficulty for the widening tunnel ID_{WT} to $ID_{\mathrm{WT}} - ID_{\mathrm{Gap}}$ and predict the *MT*.

As this study assumed that it is possible to use the navigation time in one direction to predict the navigation time in the other direction, we supposed that it is more beneficial to compute MT_{NT}, which has a longer operation time, and used the former option in the consideration in the *Discussion*.

EXPERIMENT

In order to investigate the validity of the proposed difference in difficulty, an experiment which measured *MT* over changes in the navigation direction was carried out.

Task

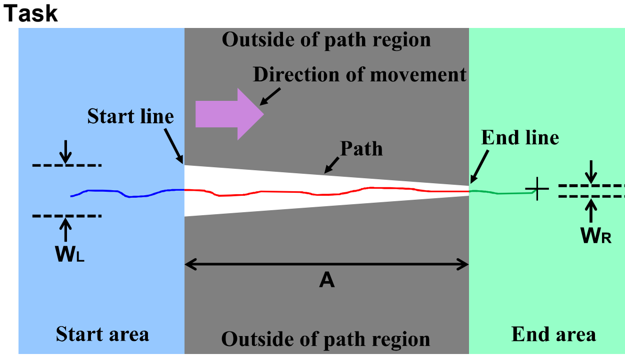


Figure 7. Screen layout of the experiment for the case of moving from left to right along a narrowing tunnel.

When the task started, a screen divided into several colors, as shown in Figure 7, was displayed on a liquid crystal pen tablet. The participant touched the stylus to the start area and then navigated past the start line as directed by the pink arrow. The participant then navigated the stylus tip all the way past the end line, all the time trying to stay within the bounds of the path. Removing the stylus from the end area then displayed a screen, which showed a selection of parameters.

When the stylus was brought close to the tablet surface, a black cross-hair cursor of 25 pixels in length was displayed. The cursor trace left a blue line when it was in the start area. Upon crossing the start line, the cursor trace turned red. Passing the end line triggered a bell to signal a successful attempt, and the cursor trace in the end area to turn green. The cursor trace turned bright orange when it was moved outside the path.

When the cursor moved outside the path or the stylus left the tablet surface, a beep sounded and the attempt was suspended. The participant put down the stylus, picked it back up, and then reentered the start area to start the test again. However, in this experiment, only the case where the cursor moved outside the path was recorded as an error. The stylus leaving the surface was regarded as a different type of error than that which was the focus of the current experiment (that is, the effect of the change in width), and was therefore not counted as an error. These are the same conditions as those of a previous study [3]. Participants were instructed to perform the required operation as quickly and accurately as possible.

Apparatus
The PC used was a Sony VAIO Z (2.1 GHz × 4 Core, 8 GB RAM, Windows 7 Pro). The display and input device was a liquid crystal pen tablet, Wacom Cintiq 12WX (IPS liquid crystal, 261.12 × 163.2 mm, 1280 × 800 pixels), with an experiment system implemented with HSP displayed in full-screen mode. The system runs at approximately 125 Hz.

Participants
Experiment participants were eleven graduate or undergraduate university students who were studying information systems (11 males, average age 21.9, $SD = 2.27$ years). All participants were right-handed and operated the stylus with that hand. One participant was very experienced in the use of the stylus, having used a pen tablet for five years and a liquid crystal pen tablet for one year. Previous research [3, 12] did not include any expert users but as the steering law held, all participants in this experiment used a stylus as well.

Procedure
The distance A was 300 and 600 pixels. The path width for both W_L and W_R was 11, 31 and 51 pixels, with only combinations where $W_L \neq W_R$ selected. The movement direction MD from start to end line was *right* and *left*. All parameters were set in reference to previous research [1, 2, 3, 8, 12]. The navigation direction {*narrowing*, *widening*} was determined according to the combination of W_R, W_L, and MD.

The total number of parameter combinations was $2(A) \times (3(W_L) \times 3(W_R) - 3$ (excluding combinations where left and right ends were the same width)) $\times 2(MD) = 24$, with the ID range between 7.47 and 31.1. These were randomly selected for six blocks of tasks, with the first block being a practice block and the remaining five blocks composing the actual tasks. The data recorded for the actual tasks totaled 24 patterns × 5 blocks × 11 participants = 1320 trials. During the practice block, the height of the seat and the angle of the tablet were adjusted. So that the participants were able to concentrate, they were given a 30-second break between each set. Total time taken was approximately 20 minutes, from preliminary instructions to completion of all tasks. After completion, participants completed a questionnaire concerning their impression of the experiment.

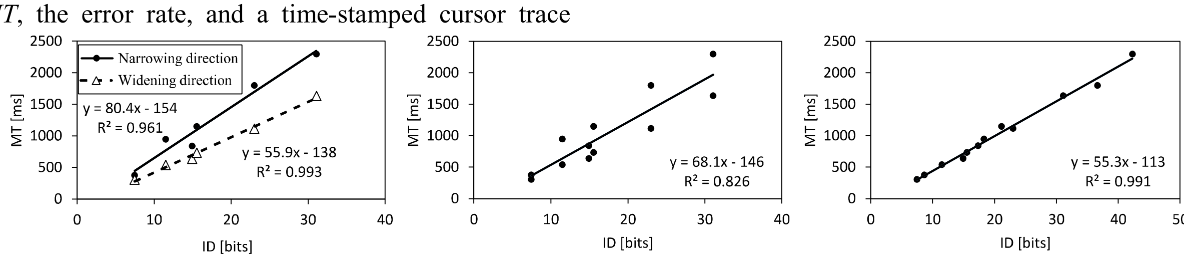
Collected Data
The time taken to pass the end line after crossing the start line MT, the error rate, and a time-stamped cursor trace were all recorded. Further, the standard deviation on the y-axis SD_y was calculated using the cursor trace data. The effectiveness of the method of analysis using the effective width [12] is the subject of much debate for the constant-width steering task, and as its applicability for paths of changing width is unclear, it was not used in this study.

Results
As it is well known that distance A [3] and width W [3, 12] cause a difference in operation time MT, this study used the ID, which combines A and W, as the independent variable which needed to be analyzed in the context of the focus of this study (i.e., effect of the navigation direction). We analyzed the data via repeated measures ANOVA and the Bonferroni post hoc test.

Operation Time MT
We observed the main effects of ID ($F_{5, 50} = 29.449$; $p < .001$) and the navigation direction ($F_{1, 10} = 23.667$; $p < .01$). A post hoc test shows that *widening* is faster than *narrowing* ($p < .01$). There is no significant effect of MD ($F_{1, 10} = 0.083$; $p = .780$). Figure 8a displays the relationship between ID and MT for both navigation directions. The plotted points represent the average of 110 trials each (2 movement directions × 5 blocks × 11 participants). The average operation time was 1233 ms for the narrowing direction and 826 ms for the widening direction.

Error Rate
We observed the main effects of ID ($F_{5, 50} = 4.204$; $p < .01$) and the navigation direction ($F_{1, 10} = 12.111$; $p < .01$). With *narrowing*, error rate becomes larger than with *widening* ($p < .01$). There is no significant effect of MD ($F_{1, 10} = 0.040$; $p = .846$). Figure 9 shows the error rate versus the ID. The average error rate for the narrowing tunnel was 6.78% (48/708), and 1.05% (7/667) for the widening tunnel. Accordingly, even for identical paths, it was found that operational mistakes increased for the narrowing direction.

Standard Deviation SD_y of the Y-axis Coordinates
We observed the main effects of ID ($F_{5, 50} = 18.208$; $p < .001$), MD ($F_{1, 10} = 11.017$; $p < .01$), and the navigation direction ($F_{1, 10} = 11.176$; $p < .01$). With $MD = left$, SD_y becomes larger than with *right* ($p < .01$). With *widening*, SD_y becomes larger than with *narrowing* ($p < .01$). Accordingly, it was found that, for identical paths, the cursor skewing was larger for movement in the widening direction.



(a) Relationship for both directions (b) Pre-correction relationship (c) Post-correction relationship

Figure 8. *MT* versus *ID*.

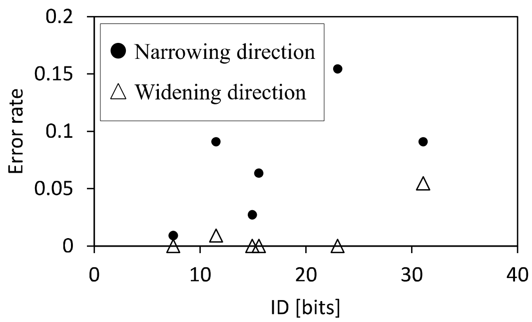


Figure 9. Error rate versus *ID*.

DISCUSSION

MT, Error Rate, and SD_y

Even for paths of exactly the same shape, the SD_y for the widening direction was larger. This means that movements with larger swings in the y-direction resulted without the cursor slipping outside of the path, and careful operation was not required. This can be thought of as being one reason for differences in *MT* occurring. In the questionnaire as well, four participants responded with comments such as, “The ones with a wider exit were easier” and “I felt that I was able to accelerate in the wider ones,” implying that they felt a change in difficulty according to the navigation direction.

Even though the SD_y for movement in the widening direction was significantly larger, the error rate for all widening *ID*s was lower (Figure 9, average value of less than 1/6 times), meaning that while the slippage was large, it did not lead to the occurrence of more errors. Summarizing the above, the *MT* for navigation in the widening direction was short, and while the SD_y was large, it was confirmed that the error rate decreased.

ID Correction

The degree of difficulty for the narrowing tunnel was corrected to $ID_{NT} + ID_{Gap}$, and we investigated whether the *MT* could be expressed through the *ID* relational equation regardless of navigation direction. In Figure 8a, an equation for the *MT* for both navigation directions was derived. The results of the regression analysis, without making a distinction for navigation direction, are shown in Figure 8b. Next, correcting the *ID* for the narrowing direction, the change in R^2 due to the *k* value is shown in Figure 10, with the greatest value of R^2 (0.991) achieved when $k = 3.14$. Substituting $k = 3.14$ into Equation 12 and deriving the ID_{Gap} yields corrected *ID*s as shown in Table 1. Figure 8c presents the relationship between *MT* and the corrected *ID*. As R^2 was extremely high (0.991), it can be stated that the relationship between the degree of difficulty *ID* attributable to the navigational direction is that expressed in Equation 12. In other words, if the *ID* for the narrowing direction is corrected with a suitable value for *k*, then it will be almost identical to the graph for the *MT* in the widening direction.

For this experiment, using the corrected *ID* for the widening direction (Figure 8a) and $k = 3.14$ in the following equation, a value that is close to the movement time in the narrowing direction can be obtained.

$$MT = -138 + 55.9 \times \left(ID + \frac{A(W_L - W_R)}{kW_L W_R}\right) \tag{13}$$

However, as it is possible that *k* will take on different values depending on the experimental conditions, it is not necessarily the case that the value calculated here (i.e., $k = 3.14$) will be the optimal value in all cases.

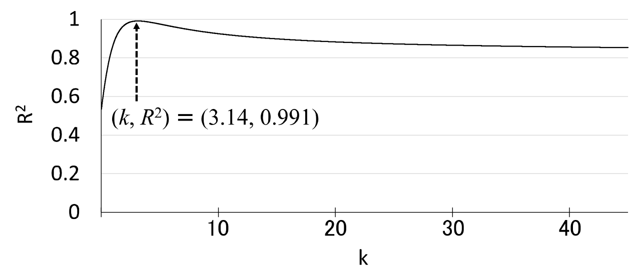


Figure 10. Model fitness depending on *k* value.

A	One *W*	The other *W*	Original *ID*	Corrected *ID*
300	31	51	7.47	8.68
300	11	51	11.5	18.3
600	31	51	14.9	17.4
300	11	31	15.5	21.1
600	11	51	23.0	36.6
600	11	31	31.1	42.3

Table 1. *ID* by the experiment parameters.

Estimating *MT* for the Case of a High *ID*

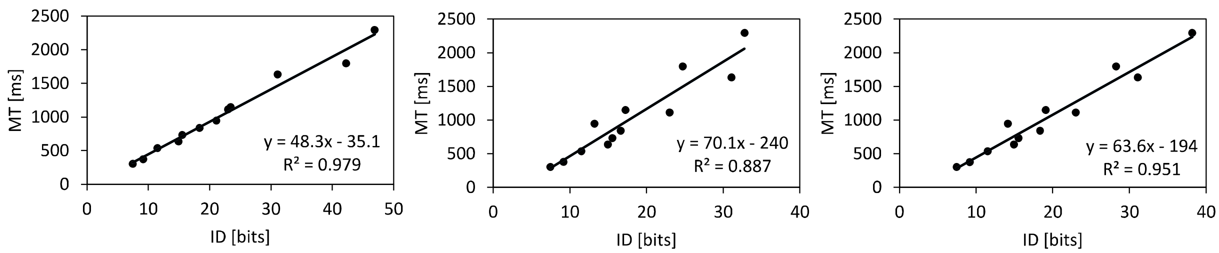
The results of the experiment showed that the model was most aligned when $k = 3.14$. Next, we will discuss the benefit of being able to predict operation time even after eliminating the experimental process. The method is as follows. First of all, measure the navigation time in the widening direction for all *ID* and then measure it in the narrowing direction for just one type of *ID*. Derive the *k*, and predict the navigation times for the other *ID*s using that *k* value.

Measurement using only the parameters of the experiment with the lowest $ID = 7.47$ (shortest recorded time) yields 375 ms, as seen in Figure 8a. The ID_{Gap} that yields $MT = 375$ in Equation 13 is:

$$375 = -138 + 55.9(7.47 + ID_{Gap}) \Leftrightarrow ID_{Gap} = 1.71$$

Deriving *k* yields:

$$1.71 = \frac{300(51-31)}{k \cdot 51 \cdot 31} \Leftrightarrow k = 2.22 \tag{14}$$



(a) After correction, using the proposed method with $k = 2.22$ (b) After correction, using a fixed value of 1.71 (c) After correction, using a fixed ratio of 1.23

Figure 11. ***MT*** **versus corrected** ***ID*****.**

Correcting the other five types of *ID* using $ID_{Gap(2.22)}$ yields the values shown in Table 2a. Figure 11a shows the relationship of the corrected *ID* and *MT*, with a high coefficient of determination $R^2 = 0.979$. Accordingly, even when carrying out the experiment with only the easiest *ID* it was confirmed that it is possible to predict the navigation time with a high degree of accuracy for non-experiment parameters.

In previous studies [1, 4], a high coefficient of determination R^2 provided the basis for claims such as the ability to explain the steering task with the model, or that for tasks crossing line segments there is a strong regularity between the operation time and *ID*. Viewing the current experiment in the same way, due to the fact that taking the optimal *k* value yields a high coefficient of determination $R^2 = 0.991$, and also that the data using the *k* value derived from the lowest *ID* also fits with an $R^2 = 0.979$, it can be said that the gap between the degree of difficulty attributable to the navigation direction can be expressed by ID_{Gap} in Equation 12.

Pre-correction *ID*	(a) Proposed Method	(b) Fixed value	(c) Fixed ratio
7.47	9.18	9.18	9.18
11.5	21.1	13.2	14.1
14.9	18.3	16.6	18.3
15.5	23.5	17.2	19.1
23.0	42.5	24.7	28.3
31.1	46.9	32.8	38.2

Table 2. ***ID*** **corrected by (a) Equation 12 with** $k = 2.22$ **(i.e., proposed method), (b) fixed value of a difference of 1.71, and (c) fixed ratio of a difference of 1.23.**

Correction Method When a Fixed-Value Difference is Assumed

Let us compare and contrast the proposed *ID* correction method with other plausible correction methods. First of all, assuming that there is simply a fixed value *ID* difference attributable to the navigation direction, the gap of difficulty for an *ID* of 7.47 will be 9.18 – 7.47 = 1.71. Adding this to the other *ID* yields corrected *ID*s as shown in Table 2b. The relationship with the *MT* is shown in Figure 11b. This does not reflect the results of the measurements in which the difference in *MT* increased as the *ID* increased. A high coefficient of determination was not able to be obtained.

Correction Method When a Fixed-Ratio Difference is Assumed

Assuming a fixed-ratio difference, the ratio for the lowest *ID* was 9.18/7.47 = 1.23. Correcting the base *ID* by a ratio of 1.23 yields the values shown in Table 2c. The relationship with *MT* is shown in Figure 11c, and a high coefficient of determination, $R^2 > 0.95$, was obtained.

Next, we investigated whether there was a difference in accuracy between the predicted time using the *ID* obtained by this correction method and the predicted time using the *ID* corrected by the proposed method. In other words, the operation time *MT* was predicted using the *ID* calculated using the *k* value of 2.22, which was derived from the lowest *ID* (Table 2a), and the *ID* calculated using the ratio derived from the same lowest *ID* (Table 2c). Then the degree to which the predicted *MT* matched the actual measured *MT* was investigated.

A Wilcoxon signed-rank test on absolute differences between observed and predicted times using each correction method {fixed ratio, proposed method} shows no significant difference between the two methods for five *ID*s ($p = .0679$), excluding *ID* = 7.47. Figure 12 shows the predicted time plotted on the horizontal axis and the observed time on the vertical axis. The nearer R^2 is to 1 the closer the predicted time is to the measured time, and the more accurate the model is. The proposed model produced a slightly higher R^2 value, but within the *ID* range of the current experiment, the method that assumed a fixed-ratio was not inferior in terms of predictive accuracy.

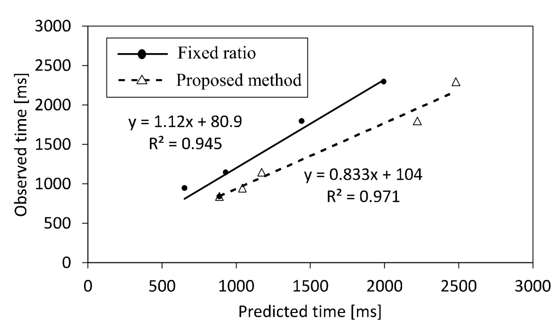


Figure 12. Observed time versus predicted time.

Observation of the Speed Profiles

Figure 13 shows the average speed profiles in the tunnels. If the current speed depends on the tunnel width of the current cursor position, the "turnover" of the speeds would appear at approximately 50% of the path length. However, we observe that speeds of the cursor in the widening direction become faster than those in the narrowing direction at approximately 25-30% for each A. This means that the speed depends not only on the current width but also on the path shape: *narrowing* or *widening*. This finding concurs with our hypothesis of Figure 5 that describes the ease with which a cursor passes through a widening tunnel rather than a narrowing tunnel. It also suggests that "an empirically determined constant" of the *local law* [1] (i.e., the instantaneous speed can be predicted by the current width and the constant) would be affected by whether the width after the current position will be narrowing or widening.

We also observe several interesting facts. First, although the start line is the easiest point to pass through a narrowing tunnel, the speeds at the start line are low for each A; users seem to gradually speed-up after entering the tunnels while they have a wide run-up area. Second, for *narrowing*, speeds increase slightly just before the goal line, despite it being the most difficult point to pass through in the tunnel. While the speeds along the widening direction show a relatively steady increase, the speeds along the narrowing direction show an initial increase, then a decreasing trend, and subsequently an increasing trend. These data suggest that users use different strategies when they pass through narrowing and widening tunnels.

Figure 14 shows single speed profiles that include clear peaks. Corrective movements (speed-up/down) did not occur at regular intervals of distance, but they occurred depending on the navigation direction and the current width; there are more peaks for *narrowing* than *widening*, and they are more often where the current width is small.

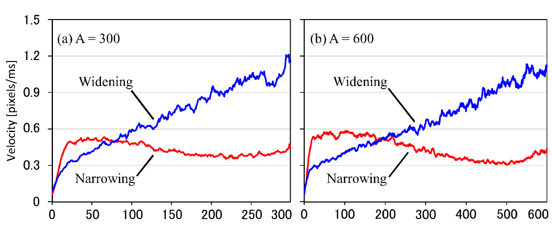


Figure 13. Average speed profiles of all the strokes filtered by a seven-point simple moving average. The left end indicates the beginning for both *narrowing* and *widening*.

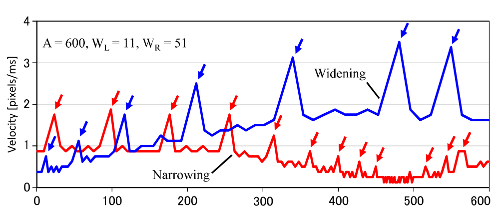


Figure 14. Single speed profiles for both directions (by the same participant, not filtered). Annotations indicate peaks.

LIMITATIONS AND FUTURE WORK

Even for identical *ID*s it is known that operation time will vary due to cursor size [16] and scale [3]. That is to say, multiple studies have shown that when experimental conditions change, it is not possible to use the *ID* in a unified way. In this paper, we have discovered that even for identical paths and cursor sizes there is another factor that can influence operation time (i.e., navigation direction: narrowing or widening). We have summarized the theoretical reasons why a difference may occur, and have confirmed through experiments that the *ID* relationship between the navigation directions is in line with the theory. The contribution of this study is to show that, based on navigation time data from one direction, it is possible to predict the time for the opposite direction with high accuracy, using only the results from a small experiment (the simplest and shortest measurement *ID*). If the navigation time for the opposite direction can be measured to a satisfactory degree, then as it is only necessary to derive the relational expression for *MT* and *ID* for each navigation direction, this study does not decrease the value of Accot and Zhai's model [1].

A limitation of this study is that high-accuracy prediction is limited to the *ID* range used in this experiment. With a higher *ID*, a prediction based on the results from a low *ID* experiment runs the risk of lowering the level of accuracy. Even though correcting with a fixed-ratio also had a high degree of accuracy–$R^2 > 0.95$, the prediction accuracy for a high *ID* is unknown in this case too. In a previous study, where an experiment was carried out on navigating a narrowing tunnel with a higher *ID*, a high coefficient of determination was found (in [1], the *ID* range was 10.9 to 76.4 with $R^2 = 0.978$). As such, it was hoped that prediction would be possible regardless of the *ID* range, but what can be emphasized from the results of this study is that for *ID*s within the range of 7.47 to 31.1, it is possible to achieve an accuracy of $R^2 = 0.971$. Future research will investigate prediction accuracy for a wider *ID* range. Further, while the optimal value of k for this experiment was $k = 3.14$, this was determined by the experiment; it is probable that the value will change depending on the experimental conditions and the user. In the same way, as the k value for the lowest *ID* (2.22) should change according to the experimental conditions, it can be presumed that it will not result in a suitable prediction of operation time if it is used as-is with different users and/or under different experiment conditions. Accordingly, it is necessary to measure the k value, which is used in the model fitting and prediction of *MT*, under various conditions. Another limitation of this study is the limited choice of parameter values, including the display size. It is necessary to test the validity of the model under the conditions of wider/narrower widths, longer/shorter amplitudes, and larger/smaller displays.

When accurate prediction is possible with fewer measured *ID* values, the benefits increase. Thus, accuracy measurement under such conditions is of interest. For example, when deriving the relational expression for *ID* and *MT* for predicting the navigation time for a narrowing tunnel, it will be possible, using the insights gained in this research, to derive it in an experiment using a minimum of two types of *ID*, in the following way. First, measure the navigation time in the widening direction for two types of *ID*, and derive the relational expression for *ID* and *MT*. Next, use the smaller *ID* to measure the navigation time in the narrowing direction. In this way, using the same procedures as in subsection *Estimating MT for the Case of a High ID*, the relational expression for *MT* and *ID* for movement in the narrowing direction can be derived. In particular, as the operation time increases when navigating in the narrowing direction with a high *ID*, the benefit of a decrease in time will occur, if it can be predicted using the data from navigation in the widening direction. Further, from the fact that there is a significant decrease in the average error rate for navigation in the widening direction, it can be hoped that the overall time required for the experiment (which is not expressed in the recorded time) can also be reduced. Note, however, that there is a concern that it may not be possible to guarantee accuracy with just two types of *ID*. Thus, there is some reason for seeking the smallest number of *ID*s that will retain an $R^2 > 0.95$, or conversely, seeking the upper limit where accuracy cannot be further improved through increasing the number of *ID*s.

While the steering law did originally have a GUI as its object of research, it has been reported that it is applicable for use in driving tasks for staying on the road in virtual reality environments [20]. That study only dealt with linear paths and rings of constant width, but by utilizing a route of non-constant width, it should be possible to design a wider variety of tasks. However, parameters are only increased to that extent and the design of the task can become complex. If the model proposed in this paper can be applied to a driving task, it can be assumed that it will be easy to set the difficulty level of all roads. For example, in the experiment in [20], single tasks were designed to take anywhere from between several tens of seconds up to one hundred seconds, but using the proposed model to predict the operation time decreases the time required to design and verify the tasks by a large amount. Other usage examples include being of help in designing a scene in a video game where the character must be steered along a long, narrow road without falling off, or without touching dangerous walls on either side of the road. This can also be thought of as being of assistance in the setting of time constraints for roads of non-constant width or in designing how much to change the difficulty level for outward and return journeys. A limitation of this study is the low number of application scenarios; additional scenarios are required to strengthen the usefulness of our proposed model. Therefore, at this stage, this study specifically focuses on the fundamental research on human behavior.

In this paper, our hypothesis was that a user makes limited number of movement corrections. However, we have not found the relationship between the number of sub-movements and the task parameters yet. Therefore, the relationship between the free weight *k* and the number of corrections is still uncertain. To avoid such a "dicey" or complex formulation, a more sophisticated (e.g., single unified) equation without a free parameter is required. An alternative approach is required to obtain such an elegant formulation. One such approach is the derivation of a feed-forward based equation: a user decides the current speed on the basis of the width of a forward position. If a user makes continuous strokes (not restricting the number of corrections) by looking slightly ahead of the tunnel, it will result in the derivation of a model significantly different from the one we propose. This hypothesis allows us to understand the reason behind the users speeding-up just before the goal line for narrowing tunnels; users aim for the wide goal area while the cursor is still on the last phase of the tunnel. In our future work, we would like to investigate such a hypothesis and validate the corresponding model.

Finally, this paper discussed results from an experiment that was carried out in a two-dimensional GUI environment using a direct-touch stylus. The question of whether the same prediction method is valid for indirect control of the cursor via a stylus [1], other devices [2], or for three-dimensional environments [7, 14], is a subject for further research. Additionally, research into whether a similar relationship to the one shown in this experiment exists for complex paths such as rings and helixes, or for paths with convex/concave interiors, and for driving simulators is planned, in order to evaluate the scope of applicability of the proposed method.

CONCLUSION
With a focus on linear tunnels with non-constant width, this paper outlined the theoretical causes of changes in the operation time *MT* due to navigation direction (narrowing or widening), and derived the *ID* difference for navigation direction. Through an experiment where a stylus was used to navigate a path in both right and left directions, the relational equation for the *MT* using the corrected *ID*, achieved a high coefficient of determination through regression analysis ($R^2 = 0.991$), and the derived *ID* difference was shown to have a certain level of validity in the experimental environment in this study. Additionally, it was shown that by measuring the navigation time for the widening direction, and the navigation time for the narrowing direction at the smallest *ID*, it is possible to predict the navigation time for the remaining *ID*s with a high degree of accuracy ($R^2 = 0.971$).

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